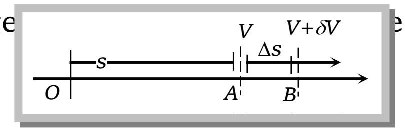
4.1 Application in Mechanics and dy/dx as a Rate Measure

4.1.1 Velocity and Acceleration in Rectilinear Motion

The velocity of a moving particle is defined as the rate of change of its displacement with respect to time and the acceleration is defined as the rate of change ect to time.



Let a particle A moves rectilinearly as shown in figure.

Let s be the displacement from a fixed point O along the path at time t; s is considered to be positive on right of the point O and negative on the left of it.

Also, Δs is positive when s increases *i.e.*, when the particle moves towards right.

Thus, if Δs be the increment in s in time Δt. The **average velocity** in this interval is $\dfrac{\Delta s}{\Delta t}$

And the instantaneous velocity *i.e.,* velocity at time t is $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

If the velocity varies, then there is change of velocity Δv in time Δt.

Hence, the acceleration at time $t = \lim\limits_{\Delta t \to 0} \dfrac{\Delta v}{\Delta t} = \dfrac{dv}{dt}$

Example: 1 The distance travelled s (in metre) by a particle in t second is given by $s = t^3 + 2t^2 + t$. The speed of the particle after 1 *sec.* will be

(a) 8 cm/sec. (b) 6 cm/sec. (c) 2 cm/sec (d) None of these

Solution: (a) $s = t^3 + 2t^2 + t$, $v = \dfrac{ds}{dt} = 3t^2 + 4t + 1$

Speed of the particle after 1 *second*

$$v_{(t=1)} = \left(\frac{ds}{dt}\right)_{(t=1)} = 3 \times 1^2 + 4 \times 1 + 1 = 8 cm/\sec.$$

Example: 2 A particle moves in a straight line in such a way that its velocity at any point is given by $v^2 = 2 - 3x$, where x is measured from a fixed point. The acceleration is

(a) Zero (b) Uniform (c) Non-uniform (d) Indeterminate

Solution: (b) Velocity, $v^2 = 2 - 3x$

Differentiating with respect to t, we get

$$2v\frac{dv}{dt} = -3.\frac{dx}{dt} \Rightarrow 2v\frac{dv}{dt} = -3v \Rightarrow \frac{dv}{dt} = -\frac{3}{2}$$

ition of a point in time 't' is given by $x = a + bt - ct^2$, $y = at + bt^2$. Its acceleration at time 't' is **[MP PET 2**
Hence, acceleration is uniform.

c (b) $(b+c)$ (c) $2b - 2c$ (d) $2\sqrt{b^2 + c^2}$

Solution: (d) Acceleration in *x*-direction = $\frac{d^2x}{dt^2} = -2c$ and acceleration in *y*-direction = $\frac{d^2y}{dt^2} = 2b$

Resultant acceleration is = $\sqrt{(-2c)^2 + (2b)^2}$ = $2\sqrt{b^2 + c^2}$

Example: 4 If the path of a moving point is the curve $x = at \quad y = b \sin at$, then its acceleration at any instant **[SCRA 1996]**

(a) Is constant (b) Varies as the distance from the axis of *x*

(c) Varies as the distance from the axis of *y* (d) Varies as the of the point from the origin

Solution: (c) $\frac{dx}{dt} = v_x = a \Rightarrow \frac{d^2x}{dt^2} = 0 = a_x$

a_x is acceleration in *x*-axis

$\frac{d^2y}{dt^2} = -ba^2 \sin at \Rightarrow a_y = -a^2 y$

Hence, a_y changes as *y* changes.

Example: 5 A stone thrown vertically upwards from the surface of the moon at velocity of 24 *m/sec*. reaches a height of $s = 24t - 0.8t^2 m$ after *t sec*. The acceleration due to gravity in m/sec^2 at the surface of the moon is **[MP PET 1992]**

(a) 0.8 (b) 1.6 (c) 2.4 (d) 4.9

Solution: (b) $\frac{ds}{dt}$ = velocity = 24 = 24 – 1.6 *t*

So acceleration at t, is $\left[\frac{d^2s}{dt^2}\right] = -1.6$

As stone is thrown upwards, so acceleration due to gravity (which acts downwards) = 1.6.

4.1.2 Derivative as the Rate of Change

If a variable quantity *y* is some function of time *t* *i.e.,* $y = f(t)$, then small change in time Δt have a corresponding change Δy in *y*.

Thus, the average rate of change = $\frac{\Delta y}{\Delta t}$

When limit $\Delta t \to 0$ is applied, the rate of change becomes instantaneous and we get the rate of change with respect to *t*.

i.e., $\lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt}$

Hence, it is clear that the rate of change of any variable with respect to some other variable is derivative of first variable with respect to other variable.

Note : ❑ The differential coefficient of *y* with respect to *x* *i.e,* $\frac{dy}{dx}$ is nothing but the rate of increase of *y* relative to *x*.

Example: 6 The rate of change of the surface area of a sphere of radius *r* when the radius is increasing at the rate of 2cm/sec is proportional to

(a) $\frac{l}{r}$ (b) $\frac{l}{r^2}$ (c) r (d) r^2

$\therefore$ — $\frac{}{dt}$ r r

$$\frac{ds}{dt} = 4\pi \times 2r\frac{dr}{} = 8\pi \times 2 = 16\pi \Rightarrow \frac{ds}{dt} \propto r.$$

Example: 7 If the volume of a spherical balloon is increasing at the rate of 900 cm^3/sec. then the rate of change of radius of balloon at instant when radius is 15 cm [in cm/sec]

(a) $\frac{22}{7}$ (b) 22 (c) $\frac{7}{22}$ (d) None of these

Solution: (c) $V = \frac{4}{3}\pi r^3$

Differentiate with respect to t

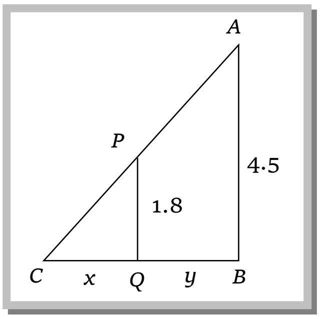
$$\frac{dV}{dt} = \frac{4}{3}\pi 3r^2 . \frac{dr}{dt} \Rightarrow \frac{dr}{dt} \Rightarrow \frac{1}{4\pi r^2} . \frac{dV}{dt}$$

$$\frac{dr}{dt} = \frac{1}{4 \times \pi \times 15 \times 15} \times 900 = \frac{1}{\pi} = \frac{7}{22}.$$

Example: 8 A man of height 1.8 m is moving away from a lamp post at the rate of 1.2 m/sec. If the height of the lamp post be 4.5 $meter$, then the rate at which the shadow of the man is lengthening

(a) 0.4 m/sec (b) 0.8 m/sec. (c) 1.2 m/sec. (d) None of these

Solution: (b) $\frac{dy}{dt} = 1.2$ According to the figure,



$x = \frac{2}{3}y$

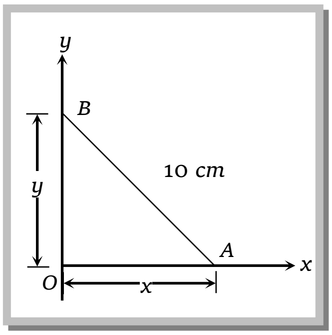
$\Rightarrow \frac{dx}{dt} = \frac{2}{3} . \frac{dy}{dt}$

$\Rightarrow$ Rate of length of shadow $\frac{dx}{dt} = 0.8\, m/s$.

Example: 9 A 10 cm long rod AB moves with its ends on two mutually perpendicular straight lines OX and OY. If the end A be moving at the rate of 2 cm/sec. then when the distance of A from O is 8 cm, the rate at which the end B is moving, is **[SCRA 1996]**

(a) $\frac{8}{3}$ cm/sec (b) $\frac{4}{3} cm/sec$ (c) $\frac{2}{9} cm/sec$. (d) None of these

Solution: (a) By figure, $x^2 + y^2 = 100$ ......(i)



$\Rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ .....(ii)

$x = 8$

Therefore by (i) and (ii), $\frac{dy}{dt} = -\frac{16}{6} = -\frac{8}{3} cm/sec$.

$\therefore$ B is moving at the rate $\frac{8}{3} cm/sec$.

Assignment

Application in Mechanics

Basic Level

1. The displacement of a particle in time t is given by $s = 2t^2 - 3t + 1$. The acceleration is

(a) 1 (b) 3 (c) 4 (d) 5

2. A stone is falling freely and describes a distance s in t seconds given by equation $s = \frac{1}{2}gt^2$. The acceleration of the stone is

(a) Uniform (b) Zero (c) Non-uniform (d) Indeterminate

3. The velocity of a particle at time t is given by the relation $v = 6t - \frac{t^2}{6}$. The distance travelled in 3 *seconds* is, if $s = 0$ at $t = 0$

(a) $\frac{39}{2}$ (b) $\frac{57}{2}$ (c) $\frac{51}{2}$ (d) $\frac{33}{2}$

4. The equation of motion of a car is $s = t^2 - 2t$, where t is measured in *hours* and s in *kilometers*. when the distance travelled by the car is 15 *km*, the velocity of the car is

(a) $2km/h$ (b) $4km/h$ (c) $6km/h$ (d) $8km/h$

5. A particle is moving in a straight line according as $s = 45t + 11t^2 - t^3$, then the time when it will come to rest, is

(a) – 9 *seconds* (b) $\frac{5}{3}$ *seconds* (c) 9 *seconds* (d) $-\frac{5}{3}$ *seconds*

6. If $t = \frac{v^2}{2}$, then $\left(-\frac{df}{dt}\right)$ is equal to (where f is acceleration) **[MP PET 1991]**

(a) f^2 (b) f^3 (c) $-f^3$ (d) $-f^2$

7. A particle is moving in a straight line according to the formula $s = t^2 + 8t + 12$. If s be measured in *meters* and t be measured in *seconds* then the average velocity of the particle in third second is

(a) 14 *m/sec* (b) 13 *m/sec* (c) 15 *m/sec* (d) None of these

8. If $2t = v^2$, then dv/dt is equal to

(a) 0 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) $\frac{1}{v}$

9. The equation of motion of a particle moving along a straight line is $s = 2t^3 - 9t^2 + 12t$, where the units of s and t are *cm* and *sec*. The acceleration of the particle will be zero after

(a) $\frac{3}{2}$ *sec* (b) $\frac{2}{3}$ *sec* (c) $\frac{1}{2}$ *sec* (d) Never

10. A body moves according to the formula $v = 1 + t^2$, where *v* is the velocity at time *t*. The acceleration after 3 *sec* will be (*v* in *cm/sec*)

[MP PET 1988]

(a) 24 cm/sec^2 (b) 12 cm/sec^2 (c) 6 cm/sec^2 (d) None of these

11. A particle moves in a straight line so that its velocity at any point is given by $v^2 = a + bx$, where $a, b \neq 0$ are constant. The acceleration is

(a) Zero (b) Uniform (c) Non-uniform (d) Indeterminate

12. The distance in seconds, described by a particle in *t* seconds is given by $s = ae^t + \frac{b}{e^t}$. The acceleration of the particle at time *t* is

(a) Proportional to *t* (b) Proportional to *s* (c) *s* (d) Constant

13. A stone thrown vertically upwards rises '*s*' *metre* in *t* seconds, where $s = 80t - 16t^2$, then velocity after 2 seconds is **[SCRA 1996]**

(a) 8 *m per sec.* (b) 16 *m per sec.* (c) 32 *m per sec.* (d) 64 *m per sec.*

14. If the distance '*s*' travelled by a particle in time *t* is $s = a \sin t + b \cos 2t$, then the acceleration at $t = 0$ is

(a) *a* (b) – *a* (c) 4*b* (d) – 4*b*

15. If the distance travelled by a point in time *t* is $s = 180t - 16t^2$, then the rate of change in velocity is

(a) – 16 *t unit* (b) 48 *unit* (c) – 32 *unit* (d) None of these

16. The motion of stone thrown up vertically is given by $s = 13.8t - 4.9t^2$, where *s* is in *metres* and *t* is in seconds. Then its velocity at $t = 1$ second is

(a) 3 *m/s* (b) 5 *m/s* (c) 4 *m/s* (d) None of these

17. A particle is moving in a straight line. Its displacement at time *t* is given by $s = -4t^2 + 2t$, then its velocity and acceleration at time $t = \frac{1}{2}$ second are

(a) –2, –8 (b) 2, 6 (c) –2, 8 (d) 2, 8

18. A ball thrown vertically upwards falls back on the ground after 6 *seconds*. Assuming that the equation of motion is of the form $s = ut - 4.9t^2$, where *s* is in *metres* and *t* is in seconds, find the velocity at $t = 0$

(a) 0 *m/s* (b) 1 *m/s* (c) 29.4 *m/s* (d) None of these

19. A particle is moving in a straight line according as $s = \sqrt{1+t}$, then the relation between its acceleration (*a*) and velocity (*v*) is

(a) $a \propto v^2$ (b) $a \propto v^3$ (c) $a \propto \frac{1}{v^3}$ (d) $a \propto v$

20. The distance travelled by a particle moving in a straight line in time *t* is $s = \sqrt{at^2 + bt + c}$. Acceleration of the particle is

[Kerala (Engg.) 2002]

(a) Proportional to *t* (b) Proportional to *s* (c) Proportional to s^{-3} (d) None of these

Advance Level

21. A particle is moving along the curve $x = at^2 + bt + c$. If $ac = b^2$, then the particle would be moving with uniform**[Orissa JE**

(a) Rotation (b) Velocity (c) Acceleration (d) Retardation

22. The equations of motion of two stones thrown vertically upwards simultaneously are $s = 19.6t - 4.9t^2$ and $s = 9.8t - 4.9t^2$ respectively and the maximum height attained by the first one is h. When the height of the first stone is maximum, the height of the second stone will be

(a) $h/3$ (b) $2h$ (c) h (d) 0

23. A particle is moving on a straight line, where its position s (in *metres*) is a function of time t (in *seconds*) given by $s = at^2 + bt + 6,\ t \geq 0$. If it is known that the particle comes to rest after 4 *seconds* at a distance of 16 *metres* from the starting position $(t = 0)$, then the retardation in its motion is

(a) $-1 m/\sec^2$ (b) $\frac{5}{4} m/\sec^2$ (c) $-\frac{1}{2} m/\sec^2$ (d) $-\frac{5}{4} m/\sec^2$

24. A point moves in a straight line during the time $t = 0$ to $t = 3$ according to the law $s = 15t - 2t^2$. The average velocity is

[MP PET 1992]

(a) 3 (b) 9 (c) 15 (d) 27

25. The equation of motion of a stone, thrown vertically upwards is $s = ut - 6.3t^2$, where the units of s and t are *cm* and *sec*. If the stone reaches at maximum height in 3*sec*. then u =

(a) 18.9 *cm/sec* (b) 12.6 *cm/sec* (c) 37.8 *cm/sec* (d) None of these

Rate Measures

Basic Level

26. Radius of a circle is increasing uniformly at the rate of 3 *cm/sec*. The rate of increase of area when radius is 10 *cm*, will be

(a) $\pi cm^2/s$ (b) $2\pi cm^2/s$ (c) $10\pi cm^2/s$ (d) None of these

27. A 10 *cm* long rod *AB* moves with its ends on two mutually perpendicular straight lines *OX* and *OY*. If the end *A* be moving at the rate of 2 *cm/sec*, then when the distance of *A* from *O* is 8 *cm*, the rate at which the end *B* is moving , is **[SCRA 1996]**

(a) $\frac{8}{3}$ *cm/sec* (b) $\frac{4}{3}$ *cm/sec* (c) $\frac{2}{9}$ *cm/sec* (d) None of these

28. If $y = x^3 + 5$ and x changes from 3 to 2.99, then the approximate change in y is

(a) 2.7 (b) – .27 (c) 27 (d) None of these

29. The volume of a spherical balloon is increasing at the rate of 40 cubic centimetre per minute. The rate of change of the surface of the balloon at the instant when its radius is 8 centimetres, is

(a) $\frac{5}{2}$ *sq cm/min.* (b) 5 *sq cm/min.* (c) 10 *sq cm/min.* (d) 20 *sq cm/min.*

30. A ladder 5 *m* in length is resting against vertical wall. The bottom of the ladder is pulled along the ground away from the wall at the rate of 1.5 *m/sec*. The length of the highest point of the ladder when the foot of the ladder is 4.0 *m* away from the wall decreases at the rate of

(a) 2 *m/sec* (b) 3 *m/sec* (c) 2.5 *m/sec* (d) 1.5 *m/sec*

31. If the rate of increase of area of a circle is not constant but the rate of increase of perimeter is constant, then the rate of increase of area varies

(a) As the square of the perimeter(b) Inversely as the perimeter (c) As the radius (d)

Advance Level

32. Gas is being pumped into a spherical balloon at the rate of 30 ft^3/min. Then the rate at which the radius increases when its reaches the value 15 *ft* is

(a) $\frac{1}{30\pi} ft/\min.$ (b) $\frac{1}{15\pi} ft/\min.$ (c) $\frac{1}{20} ft/\min.$ (d) $\frac{1}{25} ft/\min.$

33. On dropping a stone in stationary water circular ripples are observed. Rate of flow of ripples is 6 *cm*/sec. When radius of the circle is 10 *cm*, then fluid rate of increase in its area is

(a) $120\pi\, cm/\sec$ (b) $120\, sq\, cm/\sec$ (c) $\pi\, sq\, cm/\sec$ (d) $120\pi\, sq\, cm/\sec$

34. If the edge of a cube increases at the rate of 60 *cm per second,* at what rate the volume is increasing when the edge is 90 *cm*

(a) 486000 *cu cm per sec* (b) 1458000 *cu cm per sec* (c) 43740000 *cu cm per sec* (d) None of these

35. If a spherical balloon has a variable diameter $3x+\frac{9}{2}$, then the rate of change of its volume with respect to *x* is

(a) $27\pi(2x+3)^2$ (b) $\frac{27\pi}{16}(2x+3)^2$ (c) $\frac{27\pi}{8}(2x+3)^2$ (d) None of these

36. Two cyclists start from the junction of two perpendicular roads, their velocities being 3*v* metres/minute and 4*v* metres/minute. The rate at which the two cyclists are separating is

(a) $\frac{7}{2} vm/\min$ (b) $5vm/\min$ (c) $vm/\min$ (d) None of these

37. A stick of length *a cm* rests against a vertical wall and the horizontal floor. If the foot of the stick slides with a constant velocity of *b cm/s* then the magnitude of the velocity of the middle point of the stick when it is equally inclined with the floor and the wall, is

(a) $\frac{b}{\sqrt{2}} cm/s$ (b) $\frac{b}{2} cm/s$ (c) $\frac{ab}{2} cm/s$ (d) None of these

38. If $y=\int_0^x \frac{t^2}{\sqrt{t^2+1}}dt$ then the rate of change of *y* with respect to *x* when $x=1$, is

(a) $\sqrt{2}$ (b) 1/2 (c) $1/\sqrt{2}$ (d) None of these

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Answer Sheet

Assignment (Basic and Advance Level)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
c	a	c	d	c	b	b	d	a	c	b	c	b	d	c	c	a	c	b	c
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38		
c	d	b	b	c	d	a	b	c	a	c	a	d	b	c	b	a	c		